1. $\left[\frac{-1+i\sqrt{3}}{2}\right]^6+\left[\frac{-1-i\sqrt{3}}{2}\right]^6+\left[\frac{-1+i\sqrt{3}}{2}\right]^5+\left[\frac{-1-i\sqrt{3}}{2}\right]^5$ is equal to :

(A) 1 (B) – 1 (C) 2 (D) none

2. The distance between a tangent to the parabola $y^2 = 4Ax$ $(A > 0)$ and the parallel normal with gradient 1 is:

(A) 4 A (B) $2\sqrt{2}$ A (C) 2 A (D) $\sqrt{2}$ A

3. If $y = x + e^x$ then $\frac{d^2x}{dy^2}$ is :

(A) e^x (B) $-\frac{e^x}{(1+e^x)^3}$ (C) $-\frac{e^x}{(1+e^x)^2}$ (D) $\frac{-1}{(1+e^x)^3}$

4. The system of linear equations $x + y - z = 6$, $x + 2y - 3z = 14$ and $2x + 5y - \lambda z = 9$ $(\lambda \in R)$ has a unique solution if

(A) $\lambda = 8$ (B) $\lambda \neq 8$ (C) $\lambda = 7$ (D) $\lambda \neq 7$

5. If the system of equations $x + 2y + 3z = 4$, $x + py + 2z = 3$, $x + 4y + \mu z = 3$ has an infinite number of solutions, then:

(A) $p = 2, \mu = 3$ (B) $p = 2, \mu = 4$ (C) $3p = 2\mu$ (D) none of these

Space for rough work

6. Find numerically the greatest term in the expansion of $(2 + 3x)^9$, when $x = 3/2$.
(A) ${}^9C_6 . 2^9 . (3/2)^{12}$ (B) ${}^9C_3 . 2^9 . (3/2)^6$ (C) ${}^9C_5 . 2^9 . (3/2)^{10}$ (D) ${}^9C_4 . 2^9 . (3/2)^8$

7 If $\sin(xy) + \cos(xy) = 0$ then $\frac{dy}{dx} =$

(A) $\frac{y}{x}$ (B) $-\frac{y}{x}$ (C) $-\frac{x}{y}$ (D) $\frac{x}{y}$

8. If $(\sqrt{3} + i)^{100} = 2^{99}(a + ib)$, then b is equal to

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) 1 (D) none of these

9. If $(1 + x)^{10} = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{10}x^{10}$, then $(a_0 - a_2 + a_4 + a_6 + a_8 - a_{10})^2 + (a_1 - a_3 + a_5 - a_7 + a_9)^2$ is equal to
(A) 3^{10} (B) 2^{10} (C) 2^9 (D) none of these

10. If ω is a cube root of unity and $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$, then $A^{-1} =$

(A) $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}$ (B) $\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$ (D) $\frac{1}{2}\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$

Space for rough work

11. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are

(A) –2, –32 (B) –2, 3 (C) –6, 3 (D) –6, –32

12 If $x^p \cdot y^q = (x + y)^{p+q}$ then $\frac{dy}{dx}$ is :

(A) independent of p but dependent on q
(B) dependent on p but independent of q
(C) dependent on both p & q
(D) independent of p & q both .

13. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left[\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right] = 1$, then $|z_1 + z_2 + z_3|$ is :

(A) equal to 1 (B) less than 1 (C) greater than 3 (D) equal to 3

14. If $\arg(z) < 0$, then $\arg(-z) - \arg(z) =$

(A) π (B) $-\pi$ (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

Space for rough work

15. If $A\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$ and $B=\begin{bmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{bmatrix}$ then AB is equal to

(A) B (B) 3B (C) B^3 (D) A + B

16. If $f(x)=2\sin^{-1}\sqrt{1-x}+\sin^{-1}\left(2\sqrt{x(1-x)}\right)$ where $x\in\left(0,\frac{1}{2}\right)$ then f' (x) has the value equal to

(A) $\frac{2}{\sqrt{x(1-x)}}$ (B) zero (C) $-\frac{2}{\sqrt{x(1-x)}}$ (D) π

17. Let $A=\begin{bmatrix}x+\lambda&x&x\\x&x+\lambda&x\\x&x&x+\lambda\end{bmatrix}$, then A^{-1} exists if

(A) $x\neq 0$ (B) $\lambda\neq 0$ (C) $3x+\lambda\neq 0,\ \lambda\neq 0$ (D) $x\neq 0,\ \lambda\neq 0$

18. The co-efficient of x^5 in the expansion of, $(1+x)^{21}+(1+x)^{22}+\ldots\ldots+(1+x)^{30}$ is :

(A) $^{51}C_5$ (B) $^{9}C_5$ (C) $^{31}C_6-{}^{21}C_6$ (D) $^{30}C_5+{}^{20}C_5$

19 Let $f(x)=\begin{vmatrix}\cos x&\sin x&\cos x\\\cos 2x&\sin 2x&2\cos 2x\\\cos 3x&\sin 3x&3\cos 3x\end{vmatrix}$ then $f'\left(\frac{\pi}{2}\right)=$

(A) 0 (B) – 12 (C) 4 (D) 12

Space for rough work

20. If the area of the triangle included between the axes and any tangent to the curve $x^n y = a^n$ is constant, then n is equal to

(A) 1 (B) 2 (C) 3/2 (D) 1/2

Directions for Questions 21 to 23:

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, if U_1, U_2, and U_3 are columns matrices satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$. If U is 3×3 matrix whose columns are U_1, U_2, U_3 then answer the following questions

21. The value of $|U|$ is

(A) 3 (B) –3 (C) 3/2 (D) 2

22. The sum of the elements of U^{-1} is

(A) –1 (B) 0 (C) 1 (D) 3

23. The value of $[3\ 2\ 0]\ U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is

(A) 5 (B) 5/2 (C) 4 (D) 3/2

24 If $x^2y + y^3 = 2$ then the value of $\frac{d^2y}{dx^2}$ at the point (1, 1) is :

(A) $-\frac{3}{4}$ (B) $-\frac{3}{8}$ (C) $-\frac{5}{12}$ (D) none

Space for rough work

25. If $y = \frac{x^4 - x^2 + 1}{x^2 + \sqrt{3}x + 1}$ and $\frac{dy}{dx} = ax + b$ then the value of $a + b$ is equal to

(A) $\cot\frac{5\pi}{8}$ (B) $\cot\frac{5\pi}{12}$ (C) $\tan\frac{5\pi}{12}$ (D) $\tan\frac{5\pi}{8}$

26. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

27. Equation of a straight line passing through the origin and making with x – axis an angle twice the size of the angle made by the line y = 0.2 x with the x – axis, is :

(A) y = 0.4 x (B) y = (5/12) x (C) 6y – 5x = 0 (D) none of these

28. Let the co-efficients of x^n in $(1 + x)^{2n}$ & $(1 + x)^{2n-1}$ be P & Q respectively, then $\left(\frac{P+Q}{Q}\right)^5 =$

(A) 9 (B) 27 (C) 81 (D) none of these

29. If the sum of the co-efficients in the expansion of $(1 + 2x)^n$ is 6561 , then the greatest term in the expansion for x = 1/2 is :

(A) 4^{th} (B) 5^{th} (C) 6^{th} (D) none of these

Space for rough work

30. A light beam emanating from the point A(3, 10) reflects from the straight line $2x + y - 6 = 0$ and then passes through the point B(4, 3). The equation of the reflected beam is :

(A) $3x - y + 1 = 0$ (B) $x + 3y - 13 = 0$ (C) $3x + y - 15 = 0$ (D) $x - 3y + 5 = 0$

31. Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3,...... n is:

(A) $\left(\frac{n-1}{2}\right)^2$ if n is even (B) $\frac{n(n-2)}{4}$ if n is odd

(C) $\frac{(n-1)}{4}$ if n is odd (D) $\frac{n(n-2)}{4}$ if n is even

32. Area of the quadrilateral formed by the lines $|x| + |y| = 2$ is :

(A) 8 (B) 6 (C) 4 (D) none

33. Let T_r be the r^{th} term of an AP, for r = 1, 2, 3, ... If for some positive integers m, n we have $T_m = \frac{1}{n}$ & $T_n = \frac{1}{m}$, then T_{mn} equals :

(A) $\frac{1}{mn}$ (B) $\frac{1}{m} + \frac{1}{n}$ (C) 1 (D) 0

34. The number of ordered triplets of positive integers which are solutions of the equation x + y + z = 100 is:

(A) 3125 (B) 5081 (C) 6005 (D) 4851

Space for rough work

35. The equation of the bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$ which contains the points $(-1, 4)$ is :

(A) $21x + 27y - 121 = 0$
(B) $21x - 27y + 121 = 0$
(C) $21x + 27y + 191 = 0$
(D) $\frac{-3x + 4y - 12}{5} = \frac{12x - 5y + 7}{13}$

36. If $y = \cos^2(45^\circ + x) + (\sin x - \cos x)^2$ then the maximum & minimum values of y are:

(A) 2 & 0
(B) 3 & 0
(C) 3 & 1
(D) none

37. Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then :

(A) $a = \frac{7}{4}, \ r = \frac{3}{7}$
(B) $a = 2, \ r = \frac{3}{8}$
(C) $a = \frac{3}{2}, \ r = \frac{1}{2}$
(D) $a = 3, \ r = \frac{1}{4}$

38. If $\cot \alpha + \tan \alpha = m$ and $\frac{1}{\cos \alpha} - \cos \alpha = n$, then

(A) $m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1$
(B) $m(m^2n)^{1/3} - n(nm^2)^{1/3} = 1$
(C) $n(mn^2)^{1/3} - m(nm^2)^{1/3} = 1$
(D) $n(m^2n)^{1/3} \ m(mn^2)^{1/3} = 1$

Space for rough work

39 If $y = (A + Bx)\, e^{mx} + (m - 1)^{-2} e^{x}$ then $\frac{d^2y}{dx^2} - 2m\frac{dy}{dx} + m^2 y$ is equal to :

(A) e^{x} (B) e^{mx} (C) e^{-mx} (D) $e^{(1 - m)x}$

40 Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the product ab is equal to

(A) 25 (B) 9 (C) – 15 (D) – 9

41 The number of integers which lie between 1 and 10^6 and which have the sum of the digits equal to 12 is:

(A) 8550 (B) 5382 (C) 6062 (D) 8055

42 If $\sin 2\theta = k$, then the value of $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$ is equal to

(A) $\frac{1-k^2}{k}$ (B) $\frac{2-k^2}{k}$ (C) $k^2 + 1$ (D) $2 - k^2$

Space for rough work

43. The coefficient of x^{10} in the expansion of $(1 + x^2 - x^3)^8$ is
(A) 476 (B) 496 (C) 506 (D) 528

44. If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then

(A) $\alpha = \pm \frac{1}{\sqrt{2}}$ (B) $\beta = \pm \frac{1}{\sqrt{6}}$ (C) $\gamma = \pm \frac{1}{\sqrt{3}}$ (D) all of these

45. If A, B are two n × n non-singular matrices, then
(A) AB is non-singular
(B) AB is singular
(C) $(AB)^{-1} = A^{-1} B^{-1}$
(D) $(AB)^{-1}$ does not exist

46. If the tangent at each point of the curve $y = \frac{2}{3}x^3 - 2ax^2 + 2x + 5$ makes an acute angle with the positive direction of x-axis, then
(A) $a \geq 1$ (B) $-1 \leq a \leq 1$ (C) $a \leq -1$ (D) none of these

47. Equation of normal drawn to the graph of the function defined as $f(x) = \frac{\sin x^2}{x}$, $x \neq 0$ and $f(0) = 0$ at the origin is:
(A) $x + y = 0$ (B) $x - y = 0$ (C) $y = 0$ (D) $x = 0$

Space for rough work

48. The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

(A) $3\sqrt{3}$ sq. units (B) $2\sqrt{3}$ sq. units (C) $4\sqrt{3}$ sq. units (D) $\sqrt{3}$ sq. units

49. If $y = \sin^{-1}\frac{2x}{1+x^2}$ then $\left.\frac{dy}{dx}\right]_{x=-2}$ is :

(A) $\frac{2}{5}$ (B) $\frac{2}{\sqrt{5}}$ (C) $-\frac{2}{5}$ (D) none

50. The line which is parallel to x-axis and crosses the curve $y = \sqrt{x}$ at an angle of $\frac{\pi}{4}$ is

(A) $y = -1/2$ (B) $x = 1/2$ (C) $y = 1/4$ (D) $y = 1/2$

Space for rough work