I.S.C./C.B.S.E.

12

MATHEMATICS

WORKSHEETS AND ASSIGNMENTS

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CONTENTS

1 RELATIONS AND FUNCTIONS

1. Show that the relation R in the set N of Natural numbers given by
R = $\{(a, b): |a - b|\ is\ a\ multiple\ of\ 3\}$ is an equivalence relation.

2. Check whether the relation R in R defined by R = $\{(a, b): a \leq b^3\}$ is reflexive, symmetric, transitive.

3. Prove the relation R on the set N x N defined by (a, b) R (c, d) ⇔a+d=b+c , for all (a, b) (c, d) ϵ N x N is an equivalence relation.

4. Prove that the function *f* : **R**⟶ **R,** given by *f (x)* = $|x| + 5$, is not bijective.

5. Prove that the function *f* : **R**⟶ **R,** given by *f (x)* =$4x^3$ -7, is bijective

6. Prove that the Greatest Integer Function f :**R**⟶**R** given by f(x)= $[x]$, is neither one- one nor onto where $[x]$ denotes the greatest intger less than or equal to x .

7. Let *f* : **N** →**N** be defined by

f(n) = $\begin{cases} \frac{n+1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ is\ even \end{cases}$ for all $n\ \epsilon$ **N,** State whether

the function *f* is bijective.

8. Let f: **N**→ ***R*** be a function defined as f(x) = $4x^2$ +12x +15 . show that f:**N**→ ***S***, where **S** is the range of f is invertible .Find the inverse of f.

9. Consider f: **R+**→ $|[-5, \infty)$given by f(x) = $9x^2 + 6x - 5$. Show that f is invertible . Find the inverse of f.

10. Consider that f : **N** →N given by f(x) = $x^2 + x + 1$. Show that f is not invertible.

11. Let * be the binary operation on Z given by a*b = a + b-15.
1) Is * commutative ?
2) Is * associative 3) Does the identity for *exist? If yes find the identity.
4) Are the elements of Z invertible? If so find the inverse.

12. Let A=**N x N** and * be the binary operation on **A** defined by
(a,b) *(c,d) = (ad + bc, bd) .Show that * is commutative and associative .
Find the identity element for * on A , if any.

13. Determine whether the relation R defined on the set R of all real numbers as
$R = \{(a, b): a, b \in R\ and\ a - b + \sqrt{3} \in S,\ where\ S\ is\ the\ set\ of\ all\ irrational\ numbers\}$ is reflexive ,symmetric and transitive.

14. f : R→ *R* be defined by f(x) = 2x – 3 and g : R → *R* by g(x) = x^3 + 3, then find fog and show that fog is invertible. Also find $(fog)^{-1}$, hence find $(gof)^{-1}(9)$.

15. A binary operation * is defined on the set R of real numbers by
a *b = $\begin{cases} a, & if\ b = 0 \\ |a| + b, & if\ b \neq 0 \end{cases}$. If atleast one of a and b is 0,then prove that a*b = b*a. Check whether *is commutative .Find the identity element for *,if it exists.

2 INVERSE TRIGONOMETRIC FUNCTIONS

1. Find the principal value of the following:
   a) $sin^{-1}(\frac{1}{\sqrt{2}})$ b) $cos^{-1}(\frac{-1}{\sqrt{2}})$ c) $tan^{-1}(\frac{-1}{\sqrt{3}})$ d) $cosec^{-1}(-2)$ e) $sec^{-1}(-\frac{2}{\sqrt{3}})$
2. Find the value of the following:
   a) $sin^{-1}(sin\frac{3\pi}{5})$ b) $cos^{-1}(cos\frac{13\pi}{6})$ c) $tan^{-1}(tan\frac{7\pi}{6})$ d) $cosec^{-1}(cosec\frac{\pi}{8})$ e) $sec^{-1}(sec\frac{3\pi}{4})$.
3. Evaluate the following:
   a) $sin\left\{\frac{\pi}{3} - sin^{-1}\left(\frac{-1}{2}\right)\right\}$ b) $sin\left(\frac{1}{2}cos^{-1}\frac{4}{5}\right)$ c) $tan\frac{1}{2}\left(cos^{-1}\frac{\sqrt{5}}{3}\right)$.
4. Evaluate: $cos\left(sin^{-1}\frac{3}{5} + cos^{-1}\frac{12}{13}\right)$.
5. Show that $an^{-1}(\sqrt{x}) = \frac{1}{2}cos^{-1}\left(\frac{1-x}{1+x}\right)$.
6. Prove that $tan^{-1}\left\{\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right\} = \frac{\pi}{4} + \frac{1}{2}cos^{-1}x^2$.
7. Prove that $tan^{-1}\frac{1}{4} + tan^{-1}\frac{2}{9} = \frac{1}{2}cos^{-1}\frac{3}{5}$.
8. Prove that $cot^{-1}\left(\frac{ab+1}{a-b}\right) + cot^{-1}\left(\frac{bc+1}{b-c}\right) + cot^{-1}\left(\frac{ca+1}{c-a}\right) = 0$. (NCERT EXEMPLAR)
9. Prove that $tan^{-1}\frac{3}{4} + tan^{-1}\frac{3}{5} - tan^{-1}\frac{8}{19} = \frac{\pi}{4}$.
10. Prove that $cos^{-1}\frac{4}{5} + cos^{-1}\frac{12}{13} = cos^{-1}\frac{33}{65}$.
11. Solve for x: $sin^{-1}(1-x) - 2sin^{-1}x = \frac{\pi}{2}$.
12. If $cos^{-1}\frac{x}{a} + cos^{-1}\frac{y}{b} = \theta, then\ prove\ that\ \frac{x^2}{a^2} - \frac{2xy}{ab}cos\theta + \frac{y^2}{b^2} = sin^2\theta$. (NCERT EXEMPLAR)
13. Prove that $tan\left(\frac{\pi}{4} + \frac{1}{2}cos^{-1}\frac{a}{b}\right) + tan\left(\frac{\pi}{4} - \frac{1}{2}cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$. (CBSE 2010, 2013)
14. Solve for x: $cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$.
15. Solve for x: $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

3 MATRICES AND DETERMINANTS

1. Show by means of an example that the product of two non- zero matrices can be a zero matrix.

2. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = e^{ix} sinjx$.(Exemplar)

3. Solve for x and y for $x\begin{bmatrix}2\\1\end{bmatrix} + y\begin{bmatrix}3\\5\end{bmatrix} + \begin{bmatrix}-8\\-11\end{bmatrix} = 0$ (Exemplar).

4. Give an example of matrices A,B and C such that AB = AC, Where A is non-zero matrix, but B≠C.

5. Show that $A^T A \; and \; AA^T$ are both symmetric matrices for any matrix A. (Exemplar).

6. If $A = \begin{pmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{pmatrix}$ prove that $A^2 - 4A - 5I =$ Hence find A^{-1}

7. Given $A = \begin{pmatrix}3 & -4\\1 & -1\end{pmatrix}$ show by induction that $A^n = \begin{pmatrix}1+2n & -4n\\n & 1-2n\end{pmatrix}$

8. If $X = \begin{bmatrix}3 & 1 & -1\\5 & -2 & -3\end{bmatrix}$ and $Y = \begin{bmatrix}2 & 1 & -1\\7 & 2 & 4\end{bmatrix}$, Find a matrix Z such that X+Y+Z is a zero matrix. (Exemplar).

9. Find the matrix A satisfying the matrix equation :
$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix} A \begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$. (Exemplar).

10. Prove by mathematical induction that
$(A^T)^n = (A^n)^T, where\ n \in N\ for\ any\ square\ matrix\ A.$ (Exemplar).

11. If $F(\theta) = \begin{pmatrix}cos\theta & -sin\theta & 0\\sin\theta & cos\theta & 0\\0 & 0 & 1\end{pmatrix}$ show that $F(\theta)F(\varphi) = F(\theta + \varphi)$.

12. Find the inverse by elementary Operations $\begin{bmatrix}2 & -1 & 3\\1 & 3 & -1\\3 & 2 & 1\end{bmatrix}$.

13. Express the matrix $\begin{bmatrix}2 & 3 & 1\\1 & -1 & 2\\4 & 1 & 2\end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. (Exemplar).

14. Find the value of x, if

$$\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\2 & 5 & 1\\15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\2\\x\end{bmatrix} = O.$$

15) Solve the following system of linear equations:

a) 2x+ 3y +3z = 5 ; x -2y + z = -4 ; 3x – y – 2z = 3

b) $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4;\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1;\frac{6}{x}+\frac{9}{y}+\frac{20}{z}=2$

16) If $A=\begin{bmatrix}1 & -1 & 2\\ 1 & 2 & 2\\ 1 & 1 & 1\end{bmatrix}$ find A^{-1} and hence solve : x – y +2z = 9 ;x + y + z = 2 and

x + 2y +2z =3

17) Determine the product $\begin{bmatrix}-4 & 4 & 4\\ 7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and use it to solve the system of equations x-y+z=4; 2x+y+3z = 1, x – 2y -2z = 9

18) Two schools A and B want to award their selected students on the values of Tolerence,Kindness and leadership.The school P wants to award Rs x each , Rs y each and Rs z each for the three respective values to 3 , 2 and 1 students respectively with a total award money Rs 2200. School Q wants to spend Rs 3100 to award its 4,1 and 3 students on the respective values (by giving the same award money to the three values as school P) . If the total amount of award for one prize on each value is Rs 1200, using matrices ,find the award money for each value.

Apart from these three values , suggest one more value which should be considered for award.

4 DIFFERENTIATION

1.Find $\frac{dy}{dx}$ if $y = \tan^{-1}(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}})$

2.Find $\frac{dy}{dx}$, if $y = \tan^{-1}(\frac{cosx}{1+sinx})$

3.Find $\frac{dy}{dx}$, if $y = \sin^{-1}(\frac{2^{x+1}}{1+4^x})$

4. Find $\frac{dy}{dx}$, if $y = \tan^{-1}(\frac{2x}{1+15x^2})$

5. Find $\frac{dy}{dx}$, $y = (sinx)^x + \sin(x^x)$

6. $x^y = e^{y-x}$.show that dy/dx=$\frac{2-logx}{(1-logx)^2}$

7. Find $\frac{dy}{dx}$,$y = \sqrt{\frac{(x-3)(x^2+3)}{3x^2+4x+5}}$

8. Find $\frac{dy}{dx}$,$(sinx)^y = (siny)^x$.

9. Find $\frac{dy}{dx}$, $x=a(\cos\theta + \theta sin\theta)$, $y=a(\sin\theta - \theta\cos\theta)$

10. x=asin2t (1+cos2t),y=bcos2t(1-cos2t) , Showthat dy/dx at t = $\frac{\pi}{4}$ is b/a.

11. x = a(cost+tsint) , y = a(sint – tcost) . Find $\frac{d^2y}{dx^2}$.

12. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ .P.T $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

13. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}(\frac{2x}{1+x^2})$

14. If $y = [log(x + \sqrt{x^2+1})]^2$. Show that $(1+x^2)d^2y/dx^2 + x.dy/dx - 2 = 0$

15. $y = \sqrt{sinx + \sqrt{sinx + \sqrt{sinx + \ldots\ldots\ldots\ldots}}}$ P.T $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

16. Verify Rolles Theorem

$f(x) = x^3-6x^2+11x-6$ on [1 , -3]

17. Verify Rolles Theorem

$f(x) = e^x(sinx-cosx)$ on $[\frac{\pi}{4}, \frac{5\pi}{4}]$

18. Verify MVT f(x) = x (x-1)(x-2) on [0 , 1/2]

19. Discuss the applicability of Rolle's Theorem: $f(x) = \tan x$ on $[0 , \frac{\pi}{2}]$

20.If $x^m y^n = (x + y)^{m+n}$. Prove that $\frac{dy}{dx} = \frac{y}{x}$.

21. If $x^{16}y^9 = (x^2 + y)^{17}$. Prove that $\frac{dy}{dx} = \frac{2y}{x}$.

22.If $y = (x + \sqrt{x^2 + 1})]^m$, prove that $(x^2+1)\, y_2 + xy_1 - m^2y = 0$.

23.If $x = \sin(\frac{1}{a} logy)$, show that $(1 - x^2)\, y_2 - xy_1 - a^2y = 0$.

24. $f(x) = |x|^3$. Show that f is differentiable and find $f'(x)$.

25. $y=(logx)^x + x^{logx}$, find $\frac{dy}{dx}$.

26. Find the values of k ,if the following function

$$f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16}, x \neq 2 \\ k , if\ x = 2 \end{cases},\ is\ continuous\ at\ x = 2$$

27. Find the value of a and b if ,$f(x) = \begin{cases} ax^2 + b, x > 2 \\ 2 , x = 2 \\ 2ax - b,\ x < 2 \end{cases}$ is continuous at x = 2.

28. Find the value of a and b if, $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, x < \frac{\pi}{2} \\ a ,\ x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, x > \frac{\pi}{2} \end{cases}$ at $x = \frac{\pi}{2}$

29. Find the value of a , b and c if, $f(x) = \begin{cases} \frac{\sin(a+1)x+\sin x}{x}, x < 0 \\ c\ , x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}},\ x > 0 \end{cases}$ is continuous at x = 0

30. Find the value of a and b if, $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a , x < 4 \\ a + b\ , x = 4 \\ \frac{x-4}{|x-4|} + b , x > 4 \end{cases}$ is continuous at x = 4

31. Find all the points of discontinuity.

$$f(x) = \begin{cases} 2x , x < 0 \\ 0 ,0 \le x \le 1 \\ 4x , x > 1 \end{cases}$$

32. Show that $f(x) = |x + 3|$ is continuous but not differentiable at x= -3

33. Find the points where the function $f(x) = [x]$, $-2 \le x < 3$ is not differentiable.

34. Find the value of p if, $f(x) = \begin{cases} \frac{\sqrt{1+px}-\sqrt{1-px}}{x}, -1 \le x < 0 \\ \frac{2x+1}{x-2}, 0 \le x \le 1 \end{cases}$ is continuous.

5 APPLICATIONS OF DERIVATIVES

1. Sand is pouring froma pipe at the rate of $12cm^3$/sec.The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.How fast is the height of the sand-cone increasing when the height is 4cm?

2. Water is dripping out from a conical funnel at a uniform rate of $4cm^3$/sec through a tiny hole at the vertex in the bottom.When the slant height of the water is 3cm,find the rate of decrease of the slant height of the water cone .Given that the vertical angle of the funnel is 120^0.

3. Find the points on the curve $y = x^3 - 11x + 5$at which the tangent has the the equation $y = x - 1$

4. Find the equations of the tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$at the point ,where it cuts x-axis.

5. Find the points on the curve $9y^2 = x^3$ where the normal to curve makes equal intercepts with the axes.

6. Using differentials , find the approximate value of of the following upto 3 places of decimals.

a) $3.968^{3/2}$ b) $\frac{1}{\sqrt{25.1}}$

7. Find the approximate value of f(5.001) ,where $f(x)=x^3-7x^2+15$.

8. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

9. Find the intervals in which the functions given below are st.decreasing or st. increasing:-

a) $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

b) $f(x) = x^4 - \frac{x^3}{3}$

10. *Find the Intervals*in which the function f given by

$f(x)=\sin x + \cos x , 0 \leq x \leq 2\pi,$ *is* increasing or decreasing.

11. An open box with a square base is to be made out of a given quantity of metal sheet of area c^2.Show that the maximum volume of the box is $\frac{C^3}{6\sqrt{3}}$.

12. Manufacturer can sell x items at a price of rupees $(5 - \frac{x}{100})$ each . The cost price of x items is Rs $(\frac{x}{5} + 500)$. Find the number of items he should sell to earn maximum profit.

13. A point on the hypotenuse of a right angled triangle is at distance a and b from the sides .Show that the length of the hypotenuse is at least $(a^{\frac{2}{3}}+b^{\frac{2}{3}})^{3/2}$.

14. The length of the sides of an isosceles triangle are $9+x^2$, $9+x^2$ and $18-2x^2$ units. Calculate the value of x which makes the area maximum. Also find the maximum area of the triangle.

15.A window has the shape of a rectangle surmounted by an equilateral triangle . If the perimeter of the window is 12m, find the dimensions of the rectangle *that* will produce the largest area of the window.

16. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$.A soldier placed at (3 , 7) wants to shoot down the helicopter when it is nearest to him.

17. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$.

18. Show that the height of the circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h is $\frac{1}{3}h$.

19.A window of fixed perimeter (including the base of arc) is in the form of a rectangle surrounded by a semicircle .The semicircular portion is filled with coloured glass while the rectangular part is filled with clear glass.The clear glass transmits three times as much light per square metre as the coloured glass does.What is the ratio of the sides of the rectangle so that the window transits the maximum light?

6 INTEGRATION

Integrate the following:-

1. $\frac{sin^3x - cos^3 x}{sin^2x \, cos^2x}$ dx
2. $\frac{5^{(7x-5)}}{5^{(2x+10)}} \, dx$
3. $\frac{\tan x \, sec^2 x}{1-tan^2 x}$dx
4. $\frac{(x^4 - x)^{\frac{1}{4}}}{x^5}$ dx
5. $\sqrt{sin2x}$ cos2x dx
6. $\frac{\sqrt{\tan x}}{sinx \, cosx}$dx
7. $\frac{1}{1-cotx} \, dx$
8. $cosx \cos 2x \, cos3x \, dx$
9. $\frac{\sin 2x}{a^2 \, cos^2 x + b^2 \, sin^2 x}$dx
10. $tanx \, tan2x \, tan3xdx$
11. $\frac{1}{\cos(x-a)\sin(x-b}\, dx$
12. $sin^4 \, xcos^3 \, xdx$
13. $\frac{\sin\theta}{1-4cos^2\theta} \, d\theta$
14. $\frac{1}{1-6x-9x^2} \, dx$
15. $\frac{sec^2x}{5tan^2 x - 12\tan x + 14} \, dx$
16. $\frac{1}{\sqrt{9x-4x^2}} dx$
17. $\frac{1}{\sqrt{7-6x-x^2}} dx$
18. $\sqrt{1 - 4x - x^2} \, dx$
19. $\sqrt{3x + x^2 dx}$
20. $\frac{x+2}{2x^2 + 6x + 5}$ dx
21. $\frac{x+2}{\sqrt{x^2 - 1}}$
22. $\sqrt{\frac{3-x}{4+2x}} \, dx$
23. $(x+2)\sqrt{x^2 + 6x + 5} \, dx$
24. $\frac{dx}{4cos^2 x + 3 \, sin^2 x}$
25. $\frac{dx}{2cos^2 x + sin^2 x + \sin x \cos x}$
26. $\frac{x^2 + 1}{(x^2 - 5x + 6)}$

27. $\frac{\sin x}{\sin 3x}\,dx$

28. $\frac{\cos\theta}{(2+\sin\theta)(3+4\sin\theta)}$

29. $\frac{x^2+1}{(x^2+2)(2x^2+1)}\,dx$

30. $\frac{3x+5}{x^3-x^2-x+1}$

31. $\frac{x^2+x+1}{(x+1)(1+x^2)}$

32. $\frac{x^4}{(x^2+1)(x-1)}$

33. $(x\tan^{-1}x)dx$

34. $e^{3x}\cos 2x dx$

35. $e^x(\frac{1+\sin x)}{1+\cos x)}$

36. $e^x\,\frac{(x-3)e^x}{(x-1)^3}$

37. $(\frac{(x^2+1)e^x}{(x+1)^2}\,dx$

38. $\frac{x^2+4}{x^4+16}\,dx$

39. $\sqrt{\tan x\,dx}$

40. $^{\pi}_{0}\,\frac{x\tan x}{\sec x+\tan x}\,dx$

41. $^{\pi}_{0}\,\frac{x}{a^2\cos^2 x+b^2\sin^2 x}\,dx$

42. $^{\frac{\pi}{4}}_{0}\,\frac{\sin x+\cos x}{9+16\sin 2x}\,dx$

43. $^{1}_{0}\,\cot^{-1}(1-x+x^2)\,dx$

44. $^{\frac{\pi}{3}}_{\frac{\pi}{6}}\,\frac{1}{1+\sqrt{\tan x}}\,dx$

45. $^{\frac{\pi}{3}}_{\frac{\pi}{6}}\,\frac{1}{1+\sqrt{\tan x}}\,dx$

46. $^{3/2}_{-1}|x\sin(\pi x)|\,dx$

47. Find $\int_1^4 f(x)dx\ if(x)=\begin{cases}7x+3;\ if\ 1\le x\le 3\\ 8x\ if:\ 3\le x\ \le 4\end{cases}$

EVALUATE THE FOLLOWING DEFINITE INTEGRALS AS LIMIT OF SUMS

48. $^{5}_{2}(2x^2-3x+2)xdx$

49. $^{1}_{-1}e^{2x-3}\,dx$

50. $^{4}_{0}(x+e^{2x})\,dx$

7 APPLICATIONS OF INTEGRATION

1) Find the area enclosed by the parabola $y = \frac{3x^2}{4}$ *and the line* $3x - 2y + 12 = 0$.

2) Find the area of the smaller region between the ellipse $9x^2 + y^2 = 36$ and the line $\frac{x}{2} + \frac{y}{6} = 1$

3) Using integration find the area of region bounded by the triangle whose vertices are (1,0),(2,2) and (3,1).

4) Using the method of integration find the area region bounded by the lines x+2y=2,y-x=1 and 2x+y=7.

5) Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ *and* $(x-2)^2 + y^2 = 4$

6) Find the area of the region bounded by $\{(x, y): x^2 \leq y \leq |x\ \}$

7) Find the area of the region bounded the curve y = $\sqrt{1-x^2}$, line y = x and the positive x- axis.

8) Using integration ,find the area of the following region: {(x,y):$|x-1| \leq y \leq \sqrt{5-x^2}$}

9) Find the area of the region bounded the curve y =4x - x^2 and the x -axis.

10)Find the area of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$

11)Find the area of the region $\{(x, y): x^2 + y^2 \leq 8x, y^2 \geq 4x; x \geq 0; y \geq 0\}$

12)Find the area bounded by the curve y = 2x-x^2 and the line y = -x .

13)Find the area bounded by the curves y = 6x – x^2 and y = x^2 – 2x.

14)Find the area bounded by the line x = 0, x = 2 and the curves y = 2^x , y = 2x – x^2.

8 DIFFERENTIAL EQUATIONS

1. What is the degree of the following differential equation?

$$5x\left(\frac{dy}{dx}\right)^2-\frac{d^2y}{dx^2}-6y=\log x$$

2. Write the degree of the differential equation

$$x^3\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^4=0$$

3. Determine the order and degree of $t^2\frac{d^2s}{dt^2}-st\frac{ds}{dt}=s$. And, also state if it is linear or non linear.
4. Determine the order and degree of the differential equation:

$$y=px+\sqrt{a^2p^2+b^2}, where\ p=\frac{dy}{dx}$$

5. Find the integrating factor for the following differential equation:

$$x\log x\frac{dy}{dx}+y=2\log x$$

6. Find the differential equation of the family of lines passing through the origin
7. Find the differential equation of all circles, which pass through the origin and whose centres lie on the Y-axis.
8. Show that the differential equation of which $y=2(x^2-1)+ce^{-x^2}$ is a solution to $\frac{dy}{dx}+2xy=4x^3$.
9. If $y.\sqrt{x^2+1}=log\left[\sqrt{x^2+1}-x\ \right]$, Show that $(x^2+1)\frac{dy}{dx}+xy+1=0$
10. Form the differential equation of the family of curves represented by the equation: $(2x+a)^2+y^2=a^2$.
11. Write the degree of the differential equation

$$x^3\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^4=0$$

12. Solve the following differential equation:

$$\cos^2 x\frac{dy}{dx}+y=\tan x$$

13. Solve the following differential equation:
$(x^2-y^2)\,dx+2xy\,dy=0$ given that $y=1$ when $x=1$

14. Find the particular solution, satisfying the given condition, for the following differential equation:

$$\frac{dy}{dx}-\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0; y=0 \text{ when } x=1$$

15. Find the particular solution of the differential equation satisfying the given conditions:

$x^2dy+(xy+y^2)dx=0$; $y=1$ when $x=1$.

16. Find the general solution of the differential equation,

$$x\log x \frac{dy}{dx} + y = \frac{2}{x} \log x$$

17. Solve the following differential equation:

$$e^x \tan y \, dx + \left(1 - e^x\right) \sec^2 y \, dy = 0$$

18. Find the particular solution of the following differential equation:

$$(x+1)\frac{dy}{dx} = 2e^{-y} - 1; \ y = 0 \text{ when } x = 0$$

19. Find the particular solution of the differential equation

$\log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that y =0 when x = 0.

20. Find the particular solution of the differential equation $x^2dy = (2xy + y^2)\,dx$, given that y= 1, when x = 1.

21. Find the particular solution of the differential equation

$\left(1+x^2\right)\frac{dy}{dx} = \left(e^{m\tan^{-1}x} - y\right)$, given that y =1 when x = 0.

9 3-D GEOMETRY

1) Find the values of p so that the line
$\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2} and \frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ *are right angles.*
2) Find the Shortest Distance Between Two Lines
$\vec{r}=\hat{\imath}+\hat{\jmath}+\hat{\lambda}(2\hat{\imath}-\hat{\jmath}+\hat{k}) and\ \vec{r}=2\hat{\imath}+\hat{\jmath}-\hat{k}+\mu\,(3\hat{\imath}-5\hat{\jmath}+2\hat{k})$
3) Find the foot of the perpendicular from the (1,2,3) on the line $\vec{r}=6\hat{\imath}+7\hat{\jmath}+7\hat{k}+\lambda(3\hat{\imath}+2\hat{\jmath}-2\hat{k}$. *Also find the image* of the point on the line
4) Find the equation of the plane through the intersection of the planes 3x –y+2z-4=0 and x +y +z-2=0 and passing through the point (2,2,1,)
5) Find the angle between the planes whose vector equations are
$\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k})=6: \vec{r}.(2\hat{\imath}+3\hat{\jmath}+\widehat{4k})=-5$
6) Find the angle between the line $\frac{x+2}{3}=\frac{2-y}{2}=\frac{z+3}{2}$ and the plane
2x +3y-z=5
7) Find the distance of a point (3,-2,1) from the plane2x - y +2z+3=0
8) Find the length and the foot of the perpendicular from the point(7,14,5) to the plane 2x +4y-z=2 , also find image point.
9) Find the equation of the plane that contains the point (-1,3,2) and perpendicular to each of the plane x + 2y – 3z = 5 and 3x + 3y -z =0.
10) Show that the lines $\vec{r}=\hat{\imath}+\hat{\jmath}-\hat{k}+\lambda(3\hat{\imath}-\hat{\jmath}) and\ \vec{r}=4\hat{\imath}-3\hat{k}+\mu(2\hat{\imath}+3\hat{k})$ are coplanar. Also find the equation of the plane containing them.
11) Find the coordinates of the point where the line through (3,-4,-5) and (2,-3,1) crosses the 2x +y+z Plane.
12) Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-1}{5}$ *and* $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing them.
13) Prove that if a plane as the intercepts a,b,c and is at a distance of p units from the origin ,then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$
14) Find the distance of the point (1,-2,3) from the plane x-y+z=5, measured along a line parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$
15) Find the distance of the point (-2,3,-4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{-2}$ measured parallel to the plane 4x +12y -3z +1=0
16) Find the shortest distance between the pairs of lines given by
$\vec{r}=\hat{\imath}+2\hat{\jmath}+3\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+4\hat{k}) and\ \vec{r}=2\hat{\imath}+4\hat{\jmath}+5\hat{k}+\mu(3\hat{\imath}+4\hat{\jmath}+5\hat{k})$

10 VECTORS

EXERCISE I

1) Find the magnitude of the following vector:- $\vec{a} = 2\hat{\imath} - 7\hat{\jmath} - 3\hat{k}$

2) Find the unit vector in the direction of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$.

3) Find the vector joining the points P(5,3,0) and Q(-1,-2,-4) Q to P.

4) Find the position vector of the midpoint of the vector joining the points P(,3-2,0) and Q(1,-1,2).

5) Find the projection of the vector $\vec{a}$=$2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

6) If a is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 8, then\ find\ |\vec{x}|$

7) Find the area of a triangle having the points A(1,2,3), B(2-1,1) and C(-1,2,3,) as its vertices.

8) Find the area of a parallelogram whose adjacent sides are determined by the vectors **a**=$\hat{\imath} - \hat{\jmath} + 3\hat{k}$ *and* $\boldsymbol{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.

9) The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ *and* $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, Find its Area.

9) Given $|\vec{a}| = 13, |\vec{b}| = 5, and\ \vec{a}.\vec{b} = 60 . find\ |\vec{a}\ x\vec{b}|$.

10) Find λ *and* μ if $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k})\ x\ (\hat{\imath} + \lambda j + \mu k) = \vec{0}$

11) If $\vec{a}$ and $\vec{b}$ are Unit vectors Inclined at an angleθ, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$.

12) If with reference to the right handed system of mutually perpendicular unit vectors $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}, \vec{\alpha} = 3\hat{\imath} - \hat{\jmath}$,$\vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$ where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

13) $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$, $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$, Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ & $\vec{c}.\vec{d} = 15$

14. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that their magnitudes are 3, 4 and 5 respectively and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

15.Find τ if the vectors $\vec{a} = \hat{\imath} + \hat{3\jmath} + \hat{k}$,$\vec{b} = \hat{2\imath} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \tau\hat{\imath} + \hat{7\jmath} + 3\hat{k}$ are coplanar.

16. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, then find $\vec{a}\ x\ \vec{b}$ and verify $\vec{a}\ x\ \vec{b}$ is perpendicular to $\vec{a}$.

17. Find the value of λ, if the points A(-1,4,-3),B(3, λ,-5), C(-3,8,-5) and D(-3,2,1) are coplanar.

EXERCISE II

Q1. Find the magnitude of each of the following vectors :-

(i) $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$ (ii) $\vec{b} = 3\hat{i} + 4\hat{j} - 3\hat{k}$ (iii) $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

Q2. Find the unit vector in the direction of :-

(i) $\vec{a} = \hat{i}3 + 4\hat{j} - 5\hat{k}$ (ii) direction of $\overrightarrow{AB}$ if A (-2, 1, 2) & B (2, -1)

Q3. Find a vector in the direction of $\vec{a} = \hat{i}6 - 2\hat{j} + 3\hat{k}$ whose magnitude is 4 units.

Q4. Find direction ratios and direction cosins of $\vec{a} = 5\hat{i} - 3\hat{j} + 4\hat{k}$

Q5. Find the angel between the vectors $\vec{a} = (3\hat{i} - 2\hat{j} + \hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Q6. Find x for which vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ + $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are perpendicular to each other.

Q7. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Q8. Find a vector with magnitude 3 units & is perpendicular to each of the vector $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$

Q9. Find ($\vec{a}x\vec{b}$) and I $\vec{a}x\vec{b}$ I if (i) $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ & $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

(ii) $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ & $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ (iii) $\vec{a} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ & $\vec{b} = 3\hat{i} + \hat{k}$

Q10. Find the area of parallelogram whose diagonal are

(i) $\vec{d}1 = 3\hat{i} + \hat{j} - 2\hat{k}$ & $\vec{d}2 = \hat{i} - 3\hat{j} + 4\hat{k}$

(ii) $\vec{d}1 = 2\hat{i} - \hat{j} + \hat{k}$ & $\vec{d}2 = 3\hat{i} + 4\hat{j} - \hat{k}$

Q11. Using Vector find area of Δ ABC if :-

(i) A (3, -1, 2), B (1, -1, -3) & C (4, -3) (ii) A(1,2,3), B(2, 5, -1), C (-1, 1, 2)

Q12. Using vector show A, B, C are collinear pts.

(ii) A (3, -5, 1), B(-1, -, 8) & C (7, -10, -6) (ii) A(6, -7, -1) B(2, -3, 1) & C (4, -5, 0)

Q13. Verify $\vec{a}x(\vec{b} + \vec{c}) + (\vec{a}x\vec{b}) + (\vec{a}x\vec{c})if$

(i) $\vec{a} = \hat{i} - \hat{j} - 3\hat{k}$, $\overline{b} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$

(ii) $\vec{a} = 4\hat{i} - \hat{j} - \hat{k}$, $\overline{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$

Q14. If I $\vec{a}$ I = 5, I $\vec{b}$ I = 13, and ($\vec{a}x\vec{b}$) = 25, find $\vec{a}$, $\vec{b}$

Q15. If I $\vec{a}$ I = 2, I $\vec{b}$ I = 7, and ($\vec{a}x\vec{b}$) = $3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

11 PROBABILITY

1) In a class, having 60% boys, 5% of boys and 10% of girls have an IQ of more than 150. A student is selected at random and found to have an IQ of more than 150. Find the probability that the selected student is a boy.
2) A pair of dice is thrown 6 times. Getting a total of 7 on the two dice is considered a success. Find the Probability of getting (i) at least 5 successes (ii) exactly 5 successes.
3) A pair of dice is thrown 4 times . If getting a doublet is considered a success, find the probability distribution of number of successes.
4) Two cards are drawn simultaneously from a well packed deck of 52 cards . Find the mean and standard deviation of the number of the kings
5) Three Bags contain balls as shown in the table below

Bag	Number of white balls	Number of black balls	Number of red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it .They Happen to be white and red. What is probability that they came from the III bag.

6) On a multiple choice examinations with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate should get four or more correct answers just by guessing.
7) Suppose that 5% of men and 0.25% of women have grey hair , A Grey Haired person is selected at random . What is the probability of this person being a male. Assume that there are equal number of males and Females.

8) A random Variables X has the following probability distribution

X	0	1	2	3	4
P(X)	0	k	2k	2k	3k

Find (i) k (ii) $P(X<4)$ (iii) $P(X \leq 2)$ (iv) $P(0<X<3)$

9) Bag 1 contains 3 red and 4 black balls and Bag 2 Contains 4 red and 5 Black balls. Two balls are transferred at random from Bag1 to Bag2. The ball so drawn is red in colour. Find the probability that the transferred balls were both black.
10) In an examination , an examinee either guesses or copies or knows the answer of multiple choice question with 4 choice. The probability that he makes a guess is 1/3 and the probability he copies is 1/6.Find the probability that his answer is correct, given that he copied it is 1/8. Find the probability that he knew the answer to the question, given that he correctly answered it.

11) By Examining the chest X ray, probability that T.B is detected when a person is actually suffering is 0.99 the probability that the doctor diagnose incorrectly that a person has TB on the basis of Xray is 0.001. In a certain city 1 in 1000 person suffers from TB. A person selected at random is diagnosed to have TB . What is the chance that he actually has TB.

12) In a group of 50 scouts in a camp, 30 are well trained in first aid, remaining are well trained in hospitality and not first aid. 2 scouts are selected at random.Find the probability distribution of the no. of selected scouts who are well trained in first aid.Find the mean of the distribution.Write one more value which is expected from a scout.

13) In a group of 30 scientists working on an experiment,20 never commit error in their wok and report results elaborately.Two scientists are selected at random .Find the probability distribution of no. of selected scientists who never commit error in work.Also find the mean of the distribution.What values are described in this question.

14) It is known that 10% of certain articles manufactured are defective. What is the probability that in random sample of 12 such articles , 9 are defective?

15) The probability of a shooter hitting a target is ¾ . How many minimum number of times must he /she fire so that the probability of hitting the target at least once is more than 0.99?

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