TEST PAPER: MATHEMATICS

Time: 80 Minutes **Class:** 12th J.E.E.

Max. Marks: 40 Marks **Date:** 8th August, 2018

Marking Scheme: *Questions carry 10 marks each. Questions have 3 subparts each. Subparts (a) and (b) carry 3 marks each and subpart (c) carries 4 marks.* ***Attempt any 4 out of 5 questions.***

Question 1:

A. Prove Snell's law for refraction of a plane wave at a rarer medium using Huygen's Principle. *(Medium 2 is optically rarer than medium 1)*

B. State any 3 properties of electric field lines using neat diagrams.

C. i. Derive the equation for law of radioactive decay: $N_t = N_0 e^{-\lambda t}$

ii. Derive the formula for half-life using the result in part (i): $T_{\frac{1}{2}} = \frac{0.693}{\lambda}$

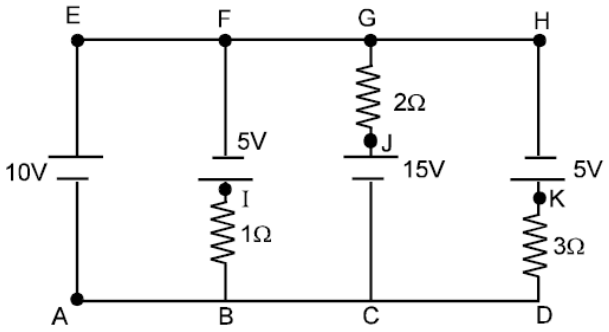
λ is the radioactive *decay constant*

Question 2:

A. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature.
What is the radius of curvature required if the focal length is to be 20cm?

B. Using Gauss's Law derive the expression for electric field (in vector form) due to a
i. Uniformly charged long wire
ii. Uniformly charged infinite sheet

C. In following circuit potential of point 'A' is zero then determine:
i. Potential of each point
ii. Potential difference across each resistance
iii. Identify the battery which act as a source
iv. Current in each battery
v. Which resistance consume maximum power
vi. Which battery consume or increased maximum energy.

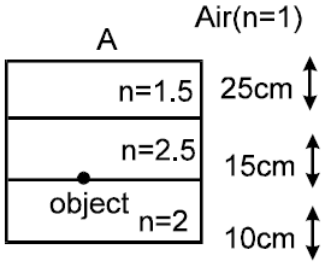


Question 3:

A. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0cm.
i. What is the magnifying power of the telescope?
ii. What is the separation between the objective and the eyepiece?

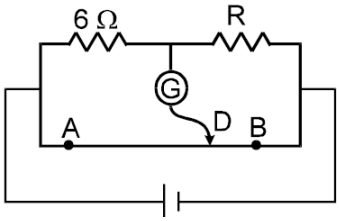
B. If the wavelength of the n^{th} line of Lyman series is equal to the De-Broglie wavelength of electron in initial orbit of a hydrogen like element (z = 11). Find the value of n.

C. Find the apparent depth of the object seen by observer A?



Question 4:

A. The meter-bridge wire AB shown in figure is 50 cm long. When AD = 30 cm, no deflection occurs in the galvanometer. Find R.



B. Using Gauss's Law derive the expression for electric field (in vector form) due to a uniformly charged solid sphere (radius R) at a point:

i. Inside the solid sphere

ii. Outside the solid sphere

C. In a double slit interference experiment, the separation between the slits is 1.0 mm, the wavelength of light used is 5.0×10^{-7} m and the distance of the screen from the slits is 1.0 m.

i. Find the distance of the centre of the first minimum from the centre of the central maximum.

ii. How many bright fringes are formed in one-centimetre width on the screen?

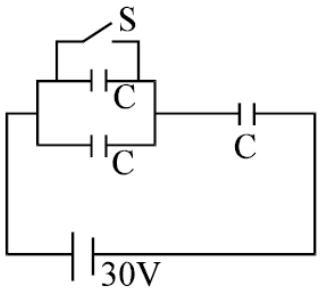
Question 5:

A. An electric field of 30 N/C exists along the negative x-axis in space. Calculate the potential difference $V_B - V_A$ where the points A and B are given by,

i. A = (0, 0) B = (0, 2m)

ii. A = (4m, 2m) B = (6m, 5m)

B. The capacitor each having capacitance C = 2μF are connected with a battery of emf 30 V as shown in figure.



When the switch S is closed. Find

i. the amount of charge flown through the battery

ii. the heat generated in the circuit

iii. the energy supplied by the battery

iv. the amount of charge flown through the switch S

C. Derive the expression for the magnitude electric field (in vector form) due to an electric dipole for a point on the dipole axis. Assume the separation of charges in dipole to be *2a*.