Mathematics
Class XII
Sample Paper – 1

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. Show that the relation R, in set of real numbers defined as $R = \{a, b : a \leq b\}$, is transitive.

2. Find the principal values of $\tan^{-1}(-1)$

3. Find the number of all possible matrices of order 3 × 3 with each entry 0 or 1.

4. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then find the position vector of their mid-point.

OR

Find the magnitude of each of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scaler product is $\frac{9}{2}$.

SECTION – B

5. Evaluate: $\int_0^p \frac{\sqrt{x}}{\sqrt{x} + \sqrt{p-x}} dx$

OR

Evaluate:

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

6. Find the area of the parallelogram having adjacent sides $\vec{a}$ and $\vec{b}$ given by $2\hat{i}+\hat{j}+k$ and $3\hat{i}+\hat{j}+4k$ respectively.

7. Find the value of $\tan(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3})$

8. Write the inverse of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

9. The contentment obtained after eating x-units of a new dish at a trial function is given by the Function C(x) = $x^3 + 6x^2 + 5x + 3$. If the marginal contentment is defined at the rate of change of (x) with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed.

10. If $e^y(x+1)=1$, show that $\frac{dy}{dx}=-e^y$.

OR

if $x = a(2\theta - \sin2\theta)$ and $y = a(1-\cos2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.

11. If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 3 and $\frac{2}{3}$ respectively such that $\vec{a}\times\vec{b}$ is a unit vector, write the angle between $\vec{a}$ and $\vec{b}$.

OR

If θ is the angle between two vectors $\hat{i}-2\hat{j}+\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$, find $\sin\theta$

12. If A is a square matrix of order 3 such that |Adj A| = 225, find |A'|.

SECTION – C

13. Show that the function

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

is continuous at $x = 0$.

14. If the sum of mean and variance of a binomial distribution for 5 trials is 1.8, find the distribution.

15. Write in the simplest form:

$$y = \cot^{-1}\left(\sqrt{1+x^2} - x\right)$$

OR

Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$

16. Let $f: N \to N$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Find whether the function f is bijective or not.

OR

Show that function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}, \forall x \in R$ is neither one-one nor onto.

Also, if $g: R \to R$ is defined as $g(x) = 2x - 1$, find fog (x)

17. If $\hat{a}$ and $\hat{b}$ are two unit vectors and θ is the angle between them, show that

$\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$.

18. Evaluate: $\int \frac{\sin x}{(1 - \cos x)(2 - \cos x)} dx$

19. Find the family of curves passing through the point (x, y) for which the slope of the tangent is equal to the sum of y-coordinate and exponential raise to the power of x-coordinate.

20. From the differential equation of the family of curves $y = A\cos 2x + B\sin 2x$, where A and B are constants.

21. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.$$

22. Find the equation of the plane passing through the points (1, 2, 3) and (0, −1, 0) and parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

23. Find the interval in which the function $f(x) = \sin^4 x + \cos^2 x$ is decreasing.

OR

Find the equation of tangent and normal to the curve $y = -3e^{5x}$ where it crosses the y-axis.

SECTION – D

24. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Find A^{-1} and hence, solve the following system of equations:

$x + 3y + 3z = 2$

$x + 4y + 3z = 1$

$x + 3y + 4z = 2$

OR

Find the inverse of the following matrix if exists, using elementary row transformation.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

25. Using integration, find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

26. A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make item A and only half an hour to make item B. The maximum time available per day is 16 hours. The profit per item of A is Rs. 300 and Rs. 160 on one item of B. How many items of each type should be produced to maximise the profit? Solve the problem graphically.

27. Find the angle between the lines whose direction cosines are given by the equations: $3l + m + 5n = 0$; $6mn - 2nl + 5lm = 0$

OR

Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) = 5$.

28. A window is in the form of a rectangle above which there is a semi-circle. If the perimeter of the window is p cm, show that the window will allow the maximum possible light only when the radius of the semi-circle is $\frac{p}{\pi + 4}$ cm.

OR

Show that the surface area of a closed cuboid with a square base and given volume is the least when it is cube.

29. In a factory which manufactures bulbs, machines X, Y and Z manufacture 1000, 2000, 3000 bulbs respectively. Of their outputs, 1%, 1.5% and 2 % are defective bulbs. A bulb is drawn at random and is found to be defective. What is the probability that the machine X manufactures it?