

**Mathematics**  
**Class XII**  
**Sample Paper - 10**

**Time: 3 hours****Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

**SECTION - A**

1. Find the value of x, if

$$\begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix} + \begin{bmatrix} 3 & x \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 10 \\ 8 & 6 \end{bmatrix}$$

2. Differentiate  $\cos\sqrt{x}$  w.r.t. x
3. Is the differential equation given by  $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y \sin y = 0$ , linear or nonlinear. Give reason.
4. Find the angle between following pairs of line

$$\frac{5-x}{-2} = \frac{y+3}{1} = \frac{1-z}{3} \text{ and } \frac{x}{3} = \frac{1-y}{-2} = \frac{z+5}{-1}$$

**OR**

Find the angle between following pairs of line

$$\frac{-x+2}{-2} = \frac{y-1}{7}, \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$$

**SECTION - B**

5. For the power set of all subsets of a non-empty set, a relation ARB is defined if and only if  $A \subset B$ . Is R an equivalence relation on the Power set?

6. Find a matrix A such that  $2A - 3B + 5C = 0$ , where  $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$  and

$$C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

7. Find :

$$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$$

- 8.

Find

$$\int \frac{dx}{5 - 8x - x^2}$$

**OR**

Evaluate:  $\int \frac{x^2 + 1}{x^4 + 1} dx$ .

9. Form the differential equation of parabolas having vertex at the origin and axis along positive y axis

10. If  $\vec{a}$  is a unit vector and  $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$  then find  $|\vec{x}|$ .

**OR**

Find the distance between the parallel planes

$$\vec{r} \cdot 2\hat{i} - 1\hat{j} + 3\hat{k} = 4 \text{ and } \vec{r} \cdot 6\hat{i} - 3\hat{j} + 9\hat{k} + 13 = 0$$

11. A company has two plants to manufacturing scooters. Plant I manufactures 70% of the scooters and plant II manufactures 30%. At plant I, 30% of the scooters are rated of standard quality and at plant II, 90% of the scooters are rated of standard quality. A scooter is chosen at random and is found to be of standard quality. Find the probability that it is manufactured by plant II.
12. In a game Rohan wins a rupee for a six and loses a rupee for any other number. Rohan has been asked to throw the dice thrice but he has to quit when he get six. Find his expected earnings.

**OR**

How many times must a fair coin be tossed so that the probability of getting atleast one head is more than 90%.

**SECTION - C**

13. A relation R on the set of complex numbers is defined by

$$z_1 R z_2 = \frac{z_1 - z_2}{z_1 + z_2}$$

Show that R is an equivalence relation.

**OR**

Let  $f: \mathbb{N} \rightarrow \mathbb{N}$  be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Find whether the function f is bijective or not.

14. Prove the following equation.

$$2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{5\sqrt{2}}{7} + 2 \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$$

15. Prove that 
$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

16. Differentiate  $\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\}$  w.r.t.  $\cos^{-1} x^2$

**OR**

If  $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ ,  $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ , find  $\frac{dy}{dx}$

17. Differentiate  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$  w.r.t.  $\tan^{-1}x$ ,  $x \neq 0$

18. Find the equation of a tangent to the curve given by  $x = a \sin^3 t$ ,  $y = b \cos^3 t$  at a point, where  $t = \frac{\pi}{2}$ .

19. Find

$$\int \frac{\cos \theta}{(4 + \sin^2 \theta)(5 - 4 \cos^2 \theta)} d\theta$$

20. Evaluate:  $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

21. Solve:  $xdy - ydx = \sqrt{x^2 + y^2} dx$

OR

Solve:  $2ye^{\frac{x}{y}} dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0$

22. Find the shortest distance between the lines  $\vec{r}_1$  and  $\vec{r}_2$  whose vector equations are

$$\vec{r}_1 = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r}_2 = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

23. Find the equation of the plane determined by the point A(3, -1, 2), B(5, 2, 4) and C(-1, -1, 6) and hence find the distance between the plane and the point P(6, 5, 9).

### SECTION - D

24. Obtain the inverse of following matrix using elementary operations

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

**OR**

If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ , then show that A is a root of polynomials  $f(x) = x^3 - 23x - 40$

**25.** Find the points of maxima and minima of the function f given by  $f(x) = (x - 2)^4(x + 1)^3$

**26.** Find the area of the smaller region bounded by the curves  $x^2 + y^2 = 4$  and  $y^2 = 3(2x - 1)$ .

**OR**

Find the area of the region bounded by lines  $y = \frac{5}{2}x - 5; x + y - 9 = 0; y = \frac{3}{4}x - \frac{3}{2}$

**27.** Find the equation of the plane through the line of intersection of the planes  $x + y + z = 1$  and  $2x + 3y + 4z = 5$  which is perpendicular to the plane  $x - y + z = 0$ . Also find the distance of the plane obtained above, from the origin.

**OR**

Find the distance of the point  $(2, 12, 5)$  from the point of intersection of the line

$$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \text{ and the plane } \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0.$$

**28.** A co-operative society of farmers has 50 hectares of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs. 10,500 and Rs. 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of Rs. 20 lakh and Rs. 10 lakh per hectare. Further, no more than Rs. 800 lakh of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society?

**29.** Two dice are rolled twice. Find the probability distribution of the random variable X, which denotes the number of doublets. Find its mean and variance.