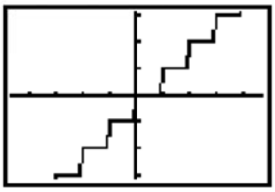
Mathematics
Class XII
Sample Paper 3

Time: **3** hour | Total Marks: 100

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. This graph does not represent a function. Make the required changes in this graph, and draw the graph, so that it represents a function.



2. Find the value of 'α' for which $\alpha(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

3. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point where the curve cuts the y-axis.

4. This 3 x 2 matrix gives information about the number of men and women workers in three factories I, II and III who lost their jobs in the last 2 months. What do you infer from the entry in the third row and second column of this matrix?

	Men workers	Women workers
Factory I	40	15
Factory II	35	40
Factory III	72	64

OR

If for any 2 x 2 square matrix A, A (adj A) = $\begin{pmatrix} 8 & 0 \\ 8 & 0 \end{pmatrix}$, then write the value of |A|.

SECTION – B

5. Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

6. Evaluate: $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right)dx$

7. For what value of 'a' the vectors $2\hat{i} - 3\hat{j} + 4k$ and $a\hat{i} + 6\hat{j} - 8k$ are collinear?

8. Write A^{-1} for $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

OR

If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

9. Simplify $\cot^{-1}\frac{-1}{\sqrt{x^2-1}}$ for $x < -1$.

10.

If $y = \tan^{-1}\frac{5x}{1-6x^2}$, $-\frac{1}{\sqrt{6}} x < -\frac{1}{\sqrt{6}}$, then prove that $\frac{dy}{dx} = \frac{2}{1+4x^2} + \frac{3}{1+9x^2}$.

OR

The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm ?

11. Obtain the differential equation of the family of circles passing through the points (a,0) and (-a,0).

12. If $P(A)=\frac{2}{5},\ P(B)=\frac{1}{3},\ \ P(A\cap B)=\frac{1}{5}$, then find $P(\bar{A}/\bar{B})$.

OR

A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "Number obtained is red." Find if A and B are independent events.

SECTION – C

13. Differentiate $\dfrac{x^3\sqrt{5+x}}{(7-3x)^5\sqrt[3]{8+5x}}$, wrt x

OR

If y = acos(log x) + bsin(log x), prove that $x^2y'' + xy' + y = 0$,

14.

Let $f(x)=x+3,\ g(x)=x-3;\ x\in N$,

Show that (i) f is not an onto function (ii) gof is an onto function

15.

Find the distance between the parallel planes

$\vec{r}.\ 2i - 1\hat{j} + 3\hat{k}\ = 4$ and $\vec{r}.\ 6\hat{i} - 3\hat{j} + 9\hat{k}\ + 13 = 0$

16.

A plane is at a distance of p units from the origin.

It makes an intercept of a,b,c with the x , y and z axis repectively.

Show that it satisfies the equation:

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$$

17. Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that

(i) All the four cards are spades?

(ii) Only 3 cards are spades?

(iii) None is a spade?

18. Solve the equation: $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\dfrac{5\pi^2}{8}$

19. Find the equation of a tangent to the curve given by $x=a\sin^3 t$, $y=b\cos^3 t$ at a point, where $t=\dfrac{\pi}{2}$.

20. Evaluate: $\int_0^{\frac{\pi}{4}}\dfrac{\sin x+\cos x}{9+16\sin 2x}dx$

21. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then prove that $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$

OR

If ω is one of the cube roots of unity, evaluate the given determinant

$$\Delta=\begin{vmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{vmatrix}$$

22. Show that the function f defined by $f(x) = |1-x+|x||$, $x \in R$ is continuous.

OR

Show that a logarithmic function is continuous at every point in its domain.

23. Evaluate: $\int\dfrac{(3\sin\alpha-2)\cos\alpha}{5-\cos^2\alpha-4\sin\alpha}d\alpha$

SECTION - D

24. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

Find the points at which the function f given by $f(x) = (x - 2)^4 (x+1)^3$ is minimum.

25. Obtain the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

26.

Calculate the area

(i) between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the x-axis between x = 0 to x = a

(ii) Triangle AOB is in the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where OA = a and OB = b.

Find the area enclosed between the chord AB and the arc AB of the ellipse

(iii) Find the ratio of the two areas found.

OR

Find the smaller of the two areas in which the circle $x^2 + y^2 = 2a^2$ is divided by the parabola $y^2 = ax$, $a > 0$

27. Find the equation of a plane that is parallel to the x-axis and passes through the line common to two intersectiing planes $\vec{r}.\ \hat{i}+\hat{j}+\hat{k}\ -1=0$ and $\vec{r}.\ 2i+3\hat{j}-\hat{k}\ =-4$

28. Two trainee carpenters A and B earn Rs. 150 and Rs. 200 per day respectively. A can make 6 frames and 4 stools per day while B can make 10 frames and 4 stools per day. How many days shall each work, if it is desired to produce atleast 60 frames and 32 stools at a minimum labour cost? Solve the problem graphically.

29. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine: (i) k (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(1 \leq X < 3)$

OR

In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses it or copies the answer. Let ½ be the probability that he knows the answer, ¼ be the probability that he guesses and ¼ be the probability that he copies it. Assuming that a student, who copies the answer, will be correct with the probability ¾, what is the probability that student knows the answer, given that he answered it correctly?

Arjun does not know that answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts of unfair means? How would an act like the above hamper his character development in the coming years?