Mathematics
Class XII
Sample Paper

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

Section-A

1. If $\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$ then find the value of x.

OR

If A is a 3 × 3 matrix |3A| = k|A|, then write the value of k.

2. Determine the value of constant k, so that the function

$$f(x) = kx^2 \quad x \le 2$$
$$= 3 \quad x > 2$$

Is continuous at x=2

3. Evaluate:

$$\int \frac{x^2}{1+x^3} dx$$

4. Write the intercept cut off by the plane 2x + y – z = 5 on x-axis.

Section-B

5. Find A^{-1} using elementary transformations

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

6. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to x-axis.

OR

Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpendicular to line $x + 14y + 3 = 0$.

7. Discuss the applicability of Lagrange's mean value theorem for the function:

$$f(x) = |\sin x| \text{ in the interval } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

8. Give the intervals in which the function $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing or decreasing.

9. Find the angle between the line

$$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6} \text{ and the plane } 10x + 2y - 11z = 3$$

10. A company has two plants to manufacturing scooters. Plant I manufactures 70% of the scooters and plant II manufactures 30%. At plant I, 30% of the scooters are rated of standard quality and at plant II, 90% of the scooters are rated of standard quality. A scooter is chosen at random and is found to be of standard quality. Find the probability that it is manufactured by plant II.

OR

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

11. A factory owner purchases two types of machines, A and B for his factory. The requirements and the limitations for the machines are as follows:

Machine	Area occupied	Labour force	Daily output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available, and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output?

12. Find

$$\int \frac{x^4 dx}{\left(x-1)(x^2+1)\right)}$$

OR

Find : $\int (x+3)\sqrt{3-4x-x^2}dx.$

Section C

13. Prove that $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\frac{\pi}{4}-\frac{x}{2}$ if $\pi<x<\frac{3\pi}{2}$

14. Find the inverse of the matrix

$A = \begin{bmatrix} 1 & 1 & 0 \\ 6 & 3 & 2 \\ 5 & 1 & 4 \end{bmatrix}$, if it exists using elementary transformations.

OR

Using the matrix method, solve the given system of equations :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4;\frac{4}{x}+\frac{-6}{y}+\frac{5}{z}=1;\frac{6}{x}+\frac{9}{y}+\frac{-20}{z}=2$$

15. Find $\frac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$.

OR

Differentiate the following function w.r.t. x:

$y = (\sin)^x + \sin^{-1}\sqrt{x}$

16. Evaluate the integral:

$$\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$$

17. Prove that,

$$\int_0^{\pi} \frac{x}{1+\sin x}dx=\pi$$

OR

Evaluate: $\int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx$.

18. Show that the differential equation $2ye^{x/y}dx + (y - 2x\,e^{x/y})\,dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that x = 0 when y = 1.

19. Given that $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$, such that the scalar product of $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and unit vector along sum of the given two vectors $\vec{b}$ and $\vec{c}$ is unity. Find this unit vector.

20. Find the vector equation of the plane passing through three points with position vectors $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$.

21. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

22. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine: (i) k (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(1 \le X < 3)$

23. A nutritionist has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. How many packet of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

Section D

24. The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result we get 7. By adding the second and third numbers to three times the first number, we get 12. Find the numbers using matrices.

25. Show that $f:[-1,1]\rightarrow R$, given by $f(x)=\frac{x}{x+2}$ is one-one. Find the inverse of the function $f:[-1,1]\rightarrow \text{Range}(f)$

26. Show that a closed right circular cylinder of a given total surface area and maximum volume is such that its height is equal to the diameter of the base.

OR

Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

27. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model A and Rs. 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

OR

A retired person wants to invest an amount of Rs. 50, 000. His broker recommends investing in two type of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least Rs. 20,000 in bond 'A' and at least Rs. 10,000 in bond 'B'. He also wants to invest at least as much in bond 'A' as in bond 'B'. Solve this linear programming problem graphically to maximise his returns.

28. Obtain the differential equation of all the circles touching the x-axis at the origin.

29. Find the equation plane passing through points (1, 2, 3), (0, -1, 0) and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$

OR

Find the equation of the plane passing through the line of intersection of the planes $2x + y - z = 3$, $5x - 3y + 4z + 9 = 0$ and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.