Mathematics
Class XII
Sample Paper -5

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

Section-A

1. If A = $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of α is A an identity matrix?

2. Determine the value of constant k, so that the function

$$f(x)=\frac{x^2-25}{x-5} \qquad x\neq 5$$
$$=k \qquad x=5$$

is continuous at x=5

OR

Determine the value of 'k' for which the following function is continuous at x =3:

$$f(x)=\begin{cases} \dfrac{(x+3)^2-36}{x-3}, & x\neq 3 \\ k, & x=3 \end{cases}$$

3. Find $\int \frac{dx}{\sqrt{9-25x^2}}$

4. Find the equation of a line through (-2, 1, 3) and parallel to $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Section-B

5. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then prove that $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$

OR

If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

6. Find the point on the curve $x^2 + y^2 - 2x - 3 = 0$, at which the tangents are parallel to x-axis

7. Find the equation of a tangent to the curve given by $x=a\sin^3 t$, $y=b\cos^3 t$ at a point, where $t=\frac{\pi}{2}$.

8. A balloon, which always remains spherical on inflation, is being by inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

OR

The total cost C(x) associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

9. Find the equation of the plane which contains the line of intersection of planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$. $\vec{r}.\left(2\hat{i}+\hat{j}+\hat{k}\right)+5=0$ and is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.

10. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "Number obtained is red." Find if A and B are independent events.

11. Find λ if the vectors $\vec{a}=\hat{i}-\lambda\hat{j}+3k$ and $\vec{b}=4\hat{i}-5\hat{j}+2k$ are perpendicular to each other.

OR

If θ is the angle between two vectors $\hat{i}-2\hat{j}+\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$, find sin θ

12. Evaluate: $\int \frac{\cos(x+a)}{\sin(x+b)}dx$

Section C

13. Solve the following equation: $\tan^{-1}\left(\frac{x-1}{x+1}\right)+\tan^{-1}\left(\frac{2x-1}{2x+1}\right)=\tan^{-1}\left(\frac{23}{36}\right)$

14. Using elementary transformations, find the inverse of the matrix $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$

OR

Using matrices solve the following system of linear equations:

$x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$

15. Using properties of determinants prove the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

OR

Find the area bounded by the curve $y = 2x - x^2$ and the line $y = -x$.

16. Evaluate: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

17. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

OR

Find $\int \frac{x^4 dx}{(x-1)(x^2+1)}$

18. Solve the given differential equation: $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$ if $y(0) = 0$

19. The vector equations of two lines are:

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} - 3\hat{j} + \hat{k})$

Find the shortest distance between the above lines.

20. Find the vector and Cartesian equation of the plane through $3\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the lines

$\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$

$\vec{r} = \hat{i} - 3\hat{j} + \hat{k} + \mu(-5\hat{i} + 4\hat{j})$

21. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

22. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.

23. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs. 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

Section D

24. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs 6,000. Three times the award money for Hard work added to that given for honesty amounts to Rs 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

25. Obtain the differential equation of all the circles touching the x-axis at the origin.

26. A nutritionist has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packet of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

OR

Find the smaller of the two areas in which the circle $x^2 + y^2 = 2a^2$ is divided by the parabola $y^2 = ax, a > 0$

27. Solve the given differential equation: $(x + 1)\dfrac{dy}{dx} = 2e^{-y} - 1$ if $y(0) = 0$

28. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Also find the distance of the plane obtained above, from the origin.

OR

Find the angle between the lines whose direction cosines are given by the equations: $3l + m + 5n = 0$; $6mn - 2nl + 5lm = 0$

29.

Let N be the set of all natural numbers and let R be the relation on $N \times N$ defined by $(a,b)\ R\ (c,d) \Leftrightarrow ad = bc$ for all $(a,b),(c,d) \in N \times N$.

Show that R is an equivalence relation on $N \times N$. Also find the equivalence class $[(2,6.)]$

OR

Consider $f : R - \left\{-\dfrac{4}{3}\right\} \to R - \left\{\dfrac{4}{3}\right\}$ given by $f(x) = \dfrac{4x+3}{3x+4}$ Show that f is bijective.

Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x)=2$