Mathematics
Class XII
Sample Paper – 6

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. A matrix has 12 elements. What are the possible orders it can have?

2. Differentiate $\sin(2x^2)$ w.r.t. x

3. Determine the order and degree of the following differential equation:
$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=e^t$$

4. Write the Cartesian equation of line passing through a point (2, -1, 4) and has direction ratios proportional to 1, 1, -2.

OR

Find the equation of a line passing through (1,-1, 0) and parallel to the line
$$\frac{x-2}{3}=\frac{2y+1}{2}=\frac{5-z}{1}$$

SECTION – B

5. (i) Is the binary operation *, defined on set N, given by
$$a*b=\frac{a+b}{2} \text{ for all } a,b\in N, \text{commutative?}$$
(ii) Is the above binary operation * associative?

6. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then find values of a and b.

7. Evaluate: $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x} \right) dx$

OR

Evaluate: $\int \frac{1 - x^2}{x(1 - 2x)} dx$

8. Evaluate: $\int \frac{2x}{\sqrt{(x^2 + 1)(x^2 + 3)}} dx$

9. Form differential equations of the family of curves represented by $c(y + c)^2 = x^3$, where c is a parameter

10. Find the angle between $\vec{a}$ and $\vec{b}$.
If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5$ & $|\vec{c}| = 7$

OR

Find λ if the vectors
$\vec{a} = \hat{i} - \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + 2\hat{k}$ are perpendicular to each other.

11. A die is tossed thrice. Find the probability of getting an odd number at least once.

OR

A random variable X has the following probability distribution. Find

X	0	1	2	3	4	5
P(X)	0.1	K	0.2	2K	0.3	K

(i) The value of K (ii) $P(X \leq 1)$ (iii) $P(X > 3)$

12. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces.

SECTION – C

13. Let A = Q × Q, where Q is the set of all rational numbers, and * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a. b), (c, d) ∈ A. Then find

(i) The identify element of * in A.

(ii) Invertible elements of A, and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$.

OR

Let $f : W \to W$
be defined as

$$f(n) = \begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$$

Show that f is invertible and find the inverse of f. Here, W is the set of all whole numbers.

14. Solve the Equation:

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x, (x > 0)$$

15. Show that x = 2 is a root of the equation formed by the following determinant

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

Hence, solve the equation.

16. If $y=\sqrt{\frac{1-x}{1+x}}$, prove that $\left(1-x^2\right)\frac{dy}{dx}+y=0$

OR

Differentiate w.r.t. x

$\log_{10}x + \log_x 10 + \log_x x + \log_{10}10$

17. Differentiate w.r.t. x $y=e^{\cos^{-1}\sqrt{1-x^2}}$

18. Find the equation of tangent and normal to the curve $y=-3e^{5x}$ where it crosses the y-axis.

19. Evaluate:

$$\int\frac{5x+3}{\sqrt{x^2+4x+10}}dx$$

20. Evaluate: $\int_0^2\left(x^2+2x+1\right)dx$ as limit of sum.

21. Solve the differential equation $(1+e^{2x})\,dy+(1+y^2)e^x\,dx=0$ given that when x = 0, y = 1.

OR

Solve the differential equation $x(1+y^2)\,dx - y(1+x^2)dy = 0$, given that y = 0, when x = 1.

22. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$

23. A variable plane is at a constant distance p from the origin and meet the coordinate axes in A, B, C. Show that the locus of the centroid of the tetrahedron OABC is
$x^{-2}+y^{-2}+z^{-2}=16p^{-2}$

SECTION – D

24. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5.

OR

Let $f(x) = x^2 - 5x + 6$. Find $f(A)$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

25. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

26. Using integration, find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

OR

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

27. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

OR

Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3k\right) - 4 = 0$, $\vec{r}.\left(2\hat{i} + \hat{j} - k\right) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.

28. A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make item A and only half an hour to make item B. The maximum time available per day is 16 hours. The profit per item of A is Rs. 300 and Rs. 160 on one item of B. How many items of each type should be produced to maximize the profit? Solve the problem graphically.

29. In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses it or copies the answer. Let ½ be the probability that he knows the answer, ¼ be the probability that he guesses and ¼ be the probability that he copies it. Assuming that a student, who copies the answer, will be correct with the probability
¾, what is the probability that student knows the answer, given that he answered it
correctly?
Arjun does not know that answer to one of the questions in the test. Which value would Arjun violate if he resorts to unfair means? How would an act like the above hamper his character development in the coming years?