Mathematics
Class XII
Sample Paper – 8

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. Write the position of the element 6 in the given matrix, and denote it as a_{ij}.

$$\begin{bmatrix} 1 & 16 & 8 & 9 \\ 7 & 5 & 3 & 2 \\ 4 & 10 & 6 & 11 \end{bmatrix}$$

2. Find $\frac{dy}{dx}$, if $2x + 3y = \cos x$

3. Is the differential equation given by $s^2\frac{d^2y}{dx^2} + sy\frac{dy}{dx} = s$, linear or nonlinear. Give reason.

4. The Cartesian equations of a line are $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Find a vector equation for the line.

OR

Find the angle between following pairs of line

$\frac{x+4}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

SECTION – B

5. Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by a * b = min {a, b}. Write the operation table of the operation *.

6. Solve the matrix equation

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

7. Evaluate:

$$\int \frac{5x-2}{1+2x+3x^2}dx$$

8. Evaluate:

$$\int \frac{x^2}{x^2+4\ \ x^2+9}dx$$

OR

Evaluate:

$$\int \frac{(x+3)e^x}{(x+5)^3}dx$$

9. Form differential equation of the family of curves y = a sin (bx + c), a and c being parameters.

10. ABCD is a parallelogram with $\overrightarrow{AB}=2\hat{i}-4\hat{j}+5\hat{k};\overrightarrow{AD}=\hat{i}-2\hat{j}-3\hat{k}$

Find a unit vector parallel to its diagonal $\vec{AC}$. Also, find the area of the parallelogram ABCD

OR

Find the projection of $(\vec{b}+\vec{c})$ on $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$.

11. Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that

(i) All the four cards are spades?
(ii) Only 3 cards are spades?
(iii) None is a spade?

12. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine: (i) k (ii) P(X < 3) (iii) P(X > 6) (iv) P(1 ≤ X < 3)

OR

A and B throw a dice alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first.

SECTION – C

13. Let A = Q x Q and let * be a binary operation on A defined by (a, b)*(c, d) = (ac, b + ad) for (a, b), (c, d) ∈ A. Determine whether * is Commutative and associative. Then, with respect to * on A

(i) Find the identify element in A.
(ii) Find the invertible elements of A.

OR

Let A = Q × Q, Q being the set of rational. Let '*' be a binary operation on A, defined by (a, b) * (c, d) = (ac, ad + b). Show that
(i) '*' is not commutative
(ii) '*' is associative
(iii) The identity element with respect to '*' is (1, 0)

14. Write in the simplest form:

$$y=\cot^{-1}\left(\sqrt{1+x^2}-x\right)$$

15. Let a, b, and c be positive numbers , but not equal and not all are zero.

Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

16. If sin y = x sin (a + y), prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

OR

If $y=b\tan^{-1}\left(\frac{x}{a}+\tan^{-1}\frac{y}{x}\right)$, find $\frac{dy}{dx}$

17. If $y=\cos^{-1}\sqrt{\frac{\cos 3x}{\cos^3 x}}$, then show that $\frac{dy}{dx}=\sqrt{\frac{3}{\cos x\cos 3x}}$

18. A man of height 2 m walks at a uniform speed of 5 km/h away from a lamp post which is 6 m high. Find the rate at which the length of his shadow increases.

19. Evaluate:

$$\int\frac{1}{\sin^4 x+\sin^2 x\cos^2 x+\cos^4 x}dx$$

20. Evaluate: $\int_{-1}^{2}(7x-5)dx$, as a limit of sums.

21. Solve the initial value problem: cos (x + y)dy = dx, y(0) = 0.

OR

Solve: $(x+y)^2\frac{dy}{dx}=a^2$

22.

a) If $\hat{i}.\hat{j}.\hat{k}$ represents the right handed system of mutually perpendicular vectors and $\vec{\alpha}=3\hat{i}-\hat{j};\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$, then express $\vec{\beta}$ in the form of $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

b) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors of magnitude 3, 4 and 5 units respectively. If each of these is perpendicular to the sum of the other two vectors, find $|\vec{a}+\vec{b}+\vec{c}|$.

23.

Find the coordinates of the point, where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersects the plane x – y + z – 5 = 0. Also find the angle between the line and the plane.

SECTION – D

24. If $A=\begin{bmatrix}1&0&2\\0&2&2\\2&0&3\end{bmatrix}$, then show that A is a root of polynomials $f(x) = x^3 - 6x^2 + 7x + 2$

OR

If $A=\begin{bmatrix}-1&-1&0\\0&-1&1\\2&3&4\end{bmatrix}$ and $B=\begin{bmatrix}1&2&3\\0&1&0\\1&1&0\end{bmatrix}$, show that $AB \neq BA$

25. Find the volume of the largest cylinder which can be inscribed in a sphere of radius r.

26. Find the area of the region
$\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$

OR

Calculate the area

(i) between the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and the x-axis between x = 0 to x = a

(ii) Triangle AOB is in the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

Where OA = a and OB = b.

Find the area enclosed between the chord AB and the arc AB of the ellipse

(iii) Find the ratio of the two areas found.

27. Find the Cartesian equation of the plane passing through the points A(0, 0, 0) and B(3, -1, 2) and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$

OR

Find the equation of the line passing through the point (-1,3,-2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.

28. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model A and Rs. 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realize a maximum profit? What is the maximum profit per week?

29. Two bags A and B contain 3 red and 4 black balls, and 4 red and 5 black balls respectively. From bag A, one ball is transferred to bag B and then a ball is drawn from bag B. The ball is found to be red in colour. Find the probability that

(a)The transferred ball is black ?

(b) The transferred ball is red ?